Rouse chain dynamics in layered random flows

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In the present paper we calculate exactly the average squared displacement (ASD) of a tagged bead of a harmonic polymer chain in a system with random, layered convection flows. We show that the ASD exhibits an anomalous behavior; its dependence on time is stronger for long than for short chains. We present also an explanation for this effect.

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I. INTRODUCTION

Much theoretical and experimental effort has been put recently in studying polymer behavior under external flows. While considerable progress has been achieved for systems in which the flows are nonrandom [1–8], the behavior of polymers in random flows is essentially less understood. The explanation of such spectacular effects as turbulent drag reduction by polymer additives still remains tentative and controversial [9–13]. This is not surprising in view of the complex physics involved and one is lead to begin with simplified models that include only some of the basic characteristics.

Here we study polymers in the presence of random convection flows; the model allows to evaluate explicitly the characteristics of polymer dynamics. The convection flows are those of the Matheron–de Marsily model [14], i.e., the flow velocities in the system are all parallel to a given axis while the direction of the flow velocity at each point depends randomly on the other coordinates (see Figs. 1 and 2). The polymer is represented as a collection of beads connected by harmonic springs and its dynamics is described by the standard Rouse model [15]. We employ this model, which is the usual starting point for the analysis of polymer dynamics in different systems, being, of course, aware of its limitations with respect to certain types of flows [6,13]. The model is also quite simplified because it disregards hydrodynamic effects [1,8]. Here we calculate exactly the time dependence of the average squared displacement (ASD) of a tagged bead of the polymer chain. The obtained result is rather surprising—we show that the ASD increases stronger with time for long than for short chains. At intermediate times, associated with the internal relaxation modes of the chain, the motion of a tagged bead has a faster time dependence than the motion of a single particle in such a system. Also at longer times the displacement of this bead is larger than the displacement of a single particle, due to the prefactor in the time dependence. We present explanations for these effects.

The paper is structured as follows: In Sec. II we define the model. Section III is devoted to the evaluation of the dynamical properties. Finally, in Sec. IV we conclude with a summary and discussion of our results.

II. THE MODEL

Consider the dynamics of a polymer chain, represented as a series of N beads linearly connected by harmonic springs, in a three-dimensional (3D) convective fluid.

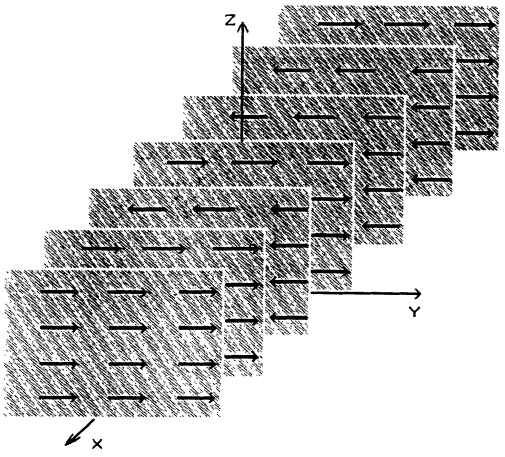

FIG. 1. Case A. The force orientation is a random function of the X variable only and is constant within each layer.

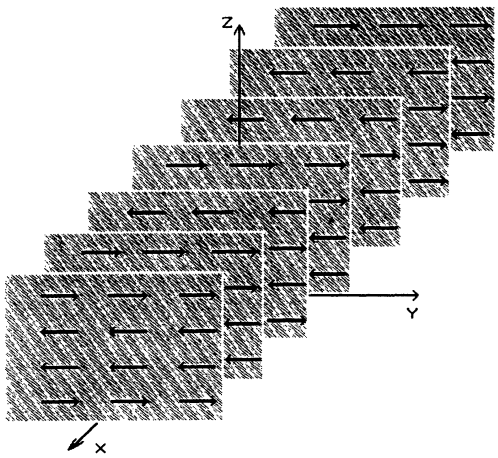

FIG. 2. Case B. The force orientation fluctuates both along the X and Z directions.

The chain's position is defined by the set $\{R_n(t)\}$, where $R_n(t)=(X_n(t),Y_n(t),Z_n(t))$ is the position vector of the nth bead at time t; $n=0,1,2,\ldots,N-1$. Ignoring excluded-volume effects and hydrodynamic interactions leads to the following system of N coupled Langevin-Rouse equations [1,8,15]

$$\zeta\frac{dR_n(t)}{dt}=-\frac{\partial U(\{R_n(t)\})}{\partial R_n(t)}+\xi_R(n,t)\ , \tag{1}$$

where ζ is the friction constant and the total elastic energy is

$$U(\{R_n(t)\})=\frac{K}{2}\sum_{n=1}^{N-1}[R_n(t)-R_{n-1}(t)]^2\ . \tag{2}$$

Here the spring constant equals $K=3T/b^2$, with T being the temperature measured in units of the Boltzmann constant K_B and b is the mean distance between beads. The random forces $\xi_R(n,t)$ in Eq. (1) mimic the collisions of the beads with the fluid molecules. It is convenient to represent the forces as a sum of two independent components

$$\xi_R(n,t)=f_R(n,t)+F(R,t)\ , \tag{3}$$

where the first term is the thermal white-noise process and the second one defines the forces induced by the convection flows. The thermal noise process is Gaussian with zero mean, so that for each component, say X,

$$\overline{f_X(n,t)}=0;\quad \overline{f_X(n,t)f_X(n',t')}=2\zeta T\delta_{n,n'}\delta(t-t')\ . \tag{4}$$

The dash denotes here and in the following thermal averages, i.e., averages with respect to the realizations of the Langevin random forces $f_R(n,t)$.

Let us discuss next the form of the convection forces $F(R,t)$. We consider quenched random layered convection fields, a model introduced by Matheron and de Marsily [14] (MdM) to describe transport in stratified porous media. The geometry in the MdM model is as follows: At any point of the system the force vector is parallel to the Y axis and the X and Z components of $F(R,t)$ are equal to zero. The Y component of $F(R,t)$ has a constant absolute value $|F(R,t)|=F_0$ and is randomly oriented along the Y axis, taking with equal probabilities the values $+F_0$ and $-F_0$. The orientation of the force vector is a function of the position in the (X,Z) plane.

Suppose that our three-dimensional (3D) system can be represented as a succession of distinct layers (Fig. 1 and 2) perpendicular to the X axis. Next, one can distinguish between two possible situations. In the first case, the force orientation is constant within each layer and varies only when going from *layer* to *layer,* i.e., is a random function of the variable X only. We will refer to this situation as case A. In this case the vector $F(R,t)$ is

$$F(R,t)=(0,F[X],0)\ . \tag{5}$$

One normally takes $F[X]$ to be a centered Gaussian random variable, with $\langle F[X]\rangle=0$ and the covariance

$$\langle F[X_1]F[X_2]\rangle=F_0\phi(|X_1-X_2|)\ . \tag{6}$$

The brackets in Eq. (6) denote averages over the stochastic state of the MdM convection fields (configurational averages) and ϕ is a given function of the distance between layers. This function defines the correlations of the flow field in the X direction. It is convenient to express it as a Fourier integral

$$\phi(|X_1-X_2|)=\int_{-\infty}^{\infty}d\omega\,Q(\omega)\exp[i\omega(X_1-X_2)]\ . \tag{7}$$

Choosing particular forms for the kernel $Q(\omega)$ we can model different types of correlations. For instance, a flat spectrum

$$Q_d(\omega)=\frac{1}{2\pi} \tag{8a}$$

corresponds to the original MdM model with δ-correlated fields, while

$$Q_{\rm lr}(\omega)=\frac{|\omega|^{\alpha-1}}{2\Gamma(\alpha)\cos(\alpha\pi/2)}\ ,\quad 0<\alpha<1\ , \tag{8b}$$

where $\Gamma(\alpha)$ is the gamma function, describes random layered fields with long-range algebraic correlations, as introduced in Ref. 16:

$$\phi_{\rm lr}(|X_1-X_2|)=\frac{1}{|X_1-X_2|^{\alpha}}\ . \tag{9}$$

In the second situation [16,17,19,20] we discuss, we let the direction of the force vector depend randomly both on X and on Z. We will call this case B and depict it in Fig. 2. Now the force vector obeys

$$F(R,t)=(0,F[X,Z],0)\ , \tag{10}$$

with $F[X,Z]$ being a Gaussian random field with moments

$$\langle F[X,Z]\rangle=0 \tag{11a}$$

and

$$\langle F[X_1,Z_1]F[X_2,Z_2]\rangle=F_0\phi(|X_1-X_2|)\eta(|Z_1-Z_2|)\ . \tag{11b}$$

Here, $\eta(|Z|)$ describes the correlations of the convection fields in the Z direction. For simplicity we will consider only the case when the correlations in the X and Z directions are characterized by the same type of spectrum, i.e., we set $\eta=\phi$.

Combining Eq. (1) with the definition of $F(R,t)$ in Eqs. (5)–(11) we get a system of equations describing the Rouse dynamics of a harmonic chain in the presence of MdM convection fields. Regarding the suffix n as a continuous variable and considering first the case A we obtain for the components of the position vectors:

$$\zeta\frac{\partial X_n(t)}{\partial t}=K\frac{\partial^2X_n(t)}{\partial n^2}+f_X(n,t)\ , \tag{12a}$$

$$\zeta\frac{\partial Z_n(t)}{\partial t}=K\frac{\partial^2Z_n(t)}{\partial n^2}+f_Z(n,t)\ , \tag{12b}$$

and

$$\zeta\frac{\partial Y_n(t)}{\partial t}=K\frac{\partial^2Y_n(t)}{\partial n^2}+F[X_n(t)]+f_Y(n,t)\ , \tag{12c}$$

In the case B the function F depends also on Z, i.e., one has $F[X_n(t),Z_n(t)]$ on the right-hand side of Eq. (12c). Finally, the Rouse boundary conditions [15] to Eqs. (12) are

$$\frac{\partial X_n(t)}{\partial n}=\frac{\partial Y_n(t)}{\partial n}=\frac{\partial Z_n(t)}{\partial n}\bigg|_{n=0,N}=0\ . \tag{13}$$

III. DYNAMICS OF THE MdM-CHAIN MODEL.

To fix the ideas we start from the dynamics of a single-particle (trivial chain with $N=1$) subject to the MdM fields. This is the original MdM model [14]. The dynamics of the particle are then described by Eqs. (12), with $K=0$. The particle undergoes a conventional diffuse motion between the layers (along the X axis) and in the Z direction, so that the X and Z components of its position vector $R(t)$ are one-dimensional Wiener processes (WP), $X(t)=1/\zeta\int^t d\tau f_X(\tau)$ and $Z(t)=1/\zeta\int^t d\tau f_Z(\tau)$. In the Y direction the particle experiences the action of the convection force superposed on the diffusive noise

$$Y(t)=\frac{1}{\zeta}\int_0^t d\tau F[X(\tau)]+\frac{1}{\zeta}\int_0^t d\tau f_Y(\tau)\ . \tag{14}$$

We note here that in the Y direction the diffusive noise produces only an additive contribution, characterized by a slower time dependence than the contribution of the convection force. To simplify matters we set the diffusive noise f_Y to zero in the rest of the paper.

The behaviors of the X and Z components are quite transparent, while the properties of the Y component of the particles's trajectory $R(t)$ are less evident. However, the nice feature of the MdM model is that the ASD along the Y axis can be computed directly. In fact the MdM model can be mapped precisely onto two well-studied problems [17]: the electron in a random potential and the Edwards self-repulsive chain. Thus, various important quantities, such as, e.g., all moments of the displacement $Y(t)$, the return probabilities and the position of the average front can be obtained explicitly [17–20]. Here we are interested only in the behavior of the second moment of $Y(t)$; for a single particle the configurational average of $Y^2(t)$ reads

$$\langle Y^2(t)\rangle=\frac{2F_0}{\zeta^2}\int_0^t d\tau_1\int_0^{\tau_1}d\tau_2\int_{-\infty}^{\infty}d\omega Q(\omega)\times\exp\{i\omega[X(\tau_1)-X(\tau_2)]\}\ . \tag{15}$$

The average with respect to the realizations of the Wiener process $X(t)$ follows readily from the evaluation of the characteristic functional:

$$\Phi(\tau_1,\tau_2;\omega)=\overline{\exp\{i\omega[X(\tau_1)-X(\tau_2)]\}}=\exp\left[-\omega^2\frac{T(\tau_1-\tau_2)}{\zeta}\right], \tag{16}$$

so that for a single particle one gets

$$\overline{\langle Y^2(t)\rangle}=\frac{2F_0}{\zeta^2}\int_0^t d\tau_1\int^{\tau_1}d\tau_2\int_{-\infty}^{\infty}d\omega\, Q(\omega)\times\exp\left[-\omega^2\frac{T(\tau_1-\tau_2)}{\zeta}\right]. \tag{17}$$

The direct computation of the integrals in Eq. (17) yields [14,16–20]

$$\overline{\langle Y^2(t)\rangle}\propto\frac{F_0}{\zeta^2}\begin{cases}D^{-1/2}t^{3/2} & \text{for } Q_d \text{ (}\delta\text{-correlated fields)} \quad (18a)\\ D^{-\alpha/2}t^{2-\alpha/2} & \text{for } Q_{\rm lr} \text{ (long range correlations)}\ . \quad (18b)\end{cases}$$

In Eqs. (18) D denotes the diffusion constant for the conventional diffusive motion in the X direction, $D=T/\zeta$.

A slightly more involved analysis performed for the case B (X-, Z-dependent fields), results in [16–20]

$$\overline{\langle Y^2(t)\rangle}\propto\frac{F_0}{\zeta^2}\begin{cases}D^{-1}t\ln(t) & \text{for } Q_d \quad (19a)\\ D^{-\alpha}t^{2-\alpha} & \text{for } Q_{\rm lr}\ . \quad (19b)\end{cases}$$

Consider next the dynamics of some tagged bead, say the nth one, of a long chain. Our primary interest is to compute the ASD along the Y axis and to see how the results in Eqs. (18) and (19) are modified due to the fact that now beads are connected in a chain. We start with the case of an infinite chain, when this effect is most pronounced, and consider finite-chains effects afterwards.

For $X_n(t)$ one gets from Eq. (12a),

$$X_n(t)=\frac{1}{\zeta}\int_0^t d\tau\int_{-\infty}^{\infty}dl\, f_X(l,\tau)P(l-n;t-\tau)\ , \tag{20}$$

where $P(l;t)$ denotes the 1D free diffusion kernel (Green's function):

$$P(l;t)=\left[\frac{\zeta}{4\pi Kt}\right]^{1/2}\exp\left[-\frac{\zeta l^2}{4Kt}\right]. \tag{21}$$

The form of $Z_n(t)$ is similar to $X_n(t)$; one has only to replace in Eq. (20) f_X by f_Z. For $Y_n(t)$ one obtains from Eq. (12c).

$$Y_n(t)=\frac{1}{\zeta}\int_0^t d\tau\int_{-\infty}^{\infty}dl\, F[X_n(\tau)]P(l-n;t-\tau)\ . \tag{22}$$

As in the single-bead case considered above, $X_n(t)$ and $Z_n(t)$ are not influenced by the convection fields and the

equation for $Y_n(t)$ is a functional of the random process $X_n(t)$ [and also of $Z_n(t)$ in the case B]. The difference between the standard MdM model of a single particle and the present situation is that for polymers the Y and X components are coupled via Eq. (22) instead of Eq. (14) and that $X_n(t)$ is not the conventional WP.

The random process $X_n(t)$ is relevant to several dynamical phenomena. For instance, Eq. (20), or, equivalently, Eq. (12a), was used to describe the motion of a tagged bead of a polymer chain in a fluid perturbed by thermal fluctuations (the original Rouse model [15]), and, also, the time evolution of the Edwards-Wilkinson [21] (EW) surface in one dimension or the formation of patterns of seggregated reactants in diffusion-controlled chemical reactions with an external input of particles [22]. In these models several types of noise processes were used (for the EW surface it was a Gaussian noise, for which the covariance decreases as a Gaussian function of the distance). The differences, however, turn out to be unimportant at large scales and at large times. Since $X_n(t)$ is a linear functional of Gaussian noise, any of its characteristics, such as, e.g., its moments to arbitrary order or the measure of trajectories [23], can be computed explicitly. These have a quite distinct form than that of the conventional WP. In particular, one can readily obtain from Eq. (20)

$$\overline{X_n^2(t)} \propto b(Dt)^{1/2} , \tag{23}$$

which differs from the WP dependence, $\overline{X^2(t)} \propto Dt$, so that the process is spatially more confined.

Let us consider next how the properties of $X_n(t)$ affect the behavior of $Y_n(t)$. The configurational average of $Y_n^2(t)$ is

$$\langle Y_n^2(t)\rangle = \frac{2F_0}{\zeta^2}\int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2 \int_{-\infty}^{\infty} dl_1 \int_{-\infty}^{\infty} dl_2 P(l_1-n,t-\tau_1)P(l_2-n,t-\tau_2)\phi[X_{l_1}(\tau_1)-X_{l_2}(\tau_2)] . \tag{24}$$

To calculate the thermal average of $\langle Y_n^2(t)\rangle$ one has to average the correlation function ϕ with respect to the realizations of the process $X_n(t)$,

$$\overline{\phi(X_{l_2}(\tau_1)-X_{l_2}(\tau_2))} = \int_{-\infty}^{\infty} d\omega Q(\omega)\overline{\exp\{i\omega[X_{l_1}(\tau_1)-X_{l_2}(\tau_2)]\}} , \tag{25}$$

which again amounts to the computation of the characteristic functional

$$\Phi(l_1,l_2;\tau_1,\tau_2;\omega)=\overline{\exp\{i\omega[X_{l_1}(\tau_1)-X_{l_2}(\tau_2)]\}} . \tag{26}$$

Now Φ is somewhat more complex since it depends also on the beads' numbers. Nonetheless, this characteristic functional can be evaluated exactly. For $\tau_2 \le \tau_1$ it equals

$$\Phi(l_1,l_2;\tau_1,\tau_2;\omega)=\exp\left[-\frac{\omega^2 T}{(\pi\zeta K)^{1/2}}M(l_1,l_2;\tau_1,\tau_2)\right] , \tag{27}$$

with

$$M(l_1,l_2;\tau_1,\tau_2)=(2\tau_1)^{1/2}+(2\tau_2)^{1/2} -4\left[\frac{\pi K}{\zeta}\right]^{1/2}\int_0^{\tau_2} d\tau P(l_1-l_2;\tau_1+\tau_2-2\tau) . \tag{28}$$

Eventually, one gets for the ASD along the Y axis

$$\overline{\langle Y_n^2(t)\rangle} = \frac{2F_0}{\zeta^2}\int_0^t d\tau_1 \int_0^{\tau_1} d\tau_2 \int_{-\infty}^{\infty} dl_1 \int_{-\infty}^{\infty} dl_2 P(l_1-n,t-\tau_1)P(l_2-n,t-\tau_2)\int_{-\infty}^{\infty} d\omega\, Q(\omega)\Phi(l_1,l_2;\tau_1,\tau_2;\omega) . \tag{29}$$

The calculation of the integrals in Eqs. (27) to (29) looks like a formidable task. However, the dependence of $\overline{\langle Y_n^2(t)\rangle}$ on the dimensional parameters and on time can be easily extracted. Using the following dimensionless variables:

$$\eta=\tau/t,\quad \eta_1=\tau_1/t,\quad \eta_2=\tau_2/t ,$$
$$p_1=(\zeta/4Kt)^{1/2}l_1,\quad p_2=(\zeta/4Kt)^{1/2}l_2 ,$$
$$\psi=T^{1/2}(t/\pi\zeta K)^{1/4}\omega , \tag{30}$$

we arrive at the following results for the behavior of the ASD in the Y direction. Omitting numerical factors, given by the dimensionless multiple integrals in Eq. (29), we get for the case A

$$\overline{\langle Y_n^2(t)\rangle} \propto \frac{F_0}{\zeta^2}\times \begin{cases} D^{-1/2}t^{3/2}(Kt/\zeta)^{1/4} & \text{for } Q_d \quad (31a)\\ D^{-\alpha/2}t^{2-\alpha/2}(Kt/\zeta)^{\alpha/4} & \text{for } Q_{\rm lr} \quad (31b)\end{cases}$$

and for the case B

$$\overline{\langle Y_n^2(t)\rangle} \propto \frac{F_0}{\zeta^2}\times \begin{cases} D^{-1}K^{1/2}t^{3/2} & \text{for } Q_d \quad (32a)\\ D^{-\alpha}t^{2-\alpha}(Kt/\zeta)^{\alpha/2} & \text{for } Q_{\rm lr} . \quad (32b)\end{cases}$$

Comparing our results Eqs. (31) and (32) with the corresponding results for a single bead, Eqs. (18) and (19), we reach the following conclusion, which at first glance seems to be rather astonishing.

In the MdM system the ASD of a tagged bead of an infinite harmonic chain grows stronger with time than the ASD of a single particle. In the case A the ASD of a bead differs from that of a single particle for δ-correlated fields by a factor of $t^{1/4}$ and for long-range correlations by a factor of $t^{\alpha/4}$. In the case B the effect of the chain is even more pronounced; the additional factors are $t^{1/2}$ and $t^{\alpha/2}$ for δ correlated and for algebraically correlated

fields, respectively.

So far we have been considering the dynamics of a tagged bead in an infinitely long chain. Let us next discuss the situation for a finite chain. The standard solution in Eqs. (12), for n defined on the interval $0 \le n \le N$, can be expressed using the normal modes [1,8,15]

$$X_n(t)=\sum_{p=-\infty}^{\infty}\cos\left[\frac{p\pi n}{N}\right]X(p,t)\ , \tag{33a}$$

$$Y_n(t)=\sum_{p=-\infty}^{\infty}\cos\left[\frac{p\pi n}{N}\right]Y(p,t) \tag{33b}$$

with

$$X(p,t)=\frac{1}{\zeta N}\int_0^t d\tau\int_0^N dn' f_X(n',\tau)\cos\left[\frac{p\pi n'}{N}\right]\times\exp\left[-\frac{p^2\pi^2K(t-\tau)}{\zeta N^2}\right] \tag{34a}$$

and

$$Y(p,t)=\frac{1}{\zeta N}\int_0^t d\tau\int_0^N dn' F[X_{n'}(\tau)]\cos\left[\frac{p\pi n'}{N}\right]\times\exp\left[\frac{-p^2\pi^2K(t-\tau)}{\zeta N^2}\right]\ . \tag{34b}$$

It is well known [1,8,15] that the behavior of the system depends on the magnitude of the time of observation t and on the so-called Rouse time $t_R=\zeta N^2/\pi^2K$; t_R defines the largest internal relaxation time of the harmonic chain. If $t \ll t_R$ the sums in Eqs. (33) can be converted into integrals, which then using Eqs. (34) leads to the representation given by Eqs. (20) and (22). Hence our results, Eqs. (31) and (32), describe in the case of a finite chain the intermediate time behavior ($t \ll t_R$) of the ASD along the Y axis. For $t \gg t_R$ the influence of the modes with $p>0$ is negligible and the motion of the bead is described only by the zeroth mode, which gives the position of the center of mass of the chain,

$$X_n(t)\propto\frac{1}{\zeta N}\int_0^t d\tau\int_0^N dn' f_X(n',\tau)\ , \tag{35a}$$

$$Y_n(t)\propto\frac{1}{\zeta N}\int_0^t d\tau\int_0^N dn' F[X_{n'}(\tau)]\ . \tag{35b}$$

Equation (35a) describes conventional diffusive motion with a, however, renormalized diffusion constant. The mean-square displacement of the tagged bead in this time regime follows $\overline{X^2(t)}=(D/N)t$, where D is the diffusion constant of a single particle. We conclude, therefore, that for $t \gg t_R$ the ASD of the bead is given by Eqs. (18) and (19) with a renormalized diffusion constant, $D \to D/N$. Explicitly

$$\langle Y_n^2(t)\rangle\propto\frac{F_0}{\zeta^2}\times\begin{cases}N^{1/2}D^{-1/2}t^{3/2} & \text{for } Q_d \\ N^{\alpha/2}D^{-\alpha/2}t^{2-\alpha/2} & \text{for } Q_{\rm lr}\ ,\end{cases} \tag{36a,36b}$$

and for the case B

$$\overline{\langle Y^2(t)\rangle}\propto\frac{F_0}{\zeta^2}\times\begin{cases}ND^{-1}t\ln(t) & \text{for } Q_d \\ N^{\alpha}D^{-\alpha}t^{2-\alpha} & \text{for } Q_{\rm lr}\ .\end{cases} \tag{37a,37b}$$

In this time regime the displacement of a tagged bead again turns out to be larger than the displacement of a single particle; the enhancement in the bead's ASD is due to the appearance of powers of N in the prefactors of Eqs. (36) and (37).

IV. DISCUSSION AND CONCLUSIONS

To summarize, we have calculated exactly the average (over the realizations of disorder) squared displacement of a tagged bead of a harmonic polymer chain in the presence of random layered convection flows. We have shown that the ASD exhibits a rather spectacular behavior: It increases more with time for long rather than for short chains.

The effect of the enhancement of the bead's ASD with the chain length has a transparent explanation and, in general, is to be expected on physical grounds. To see this, we return to the MdM model for a single particle and recall the physical mechanism responsible for anomalous results in Eqs. (18) and (19). For simplicity we consider only case A with δ-correlated fields. The particle's displacement in the Y direction, Eq. (14), can be formally rewritten as

$$Y(t)=\frac{1}{\zeta}\int_{-\infty}^{\infty}N(X,t)F[X]dX\ , \tag{38}$$

where $N(X,t)=\int_0^t dt'\delta[X-X(t')]$ is the number of times the layer at position X has been visited by the particle up to time t. Consequently, the ASD along the Y axis has the form

$$\overline{\langle Y^2(t)\rangle}=\frac{1}{\zeta^2}\int_{-\infty}^{\infty}dX\,N^2(X,t)\ . \tag{39}$$

A heuristic estimate of the ASD in Eq. (39) in the case when the particle performs a convectional diffusive motion along the X axis is as follows [18–20,24]. The number of visits of a given layer is proportional to the elapsed time t divided by the number of distinct layers encountered by the particle during t. The latter grows for conventional 1D diffusion as $(Dt)^{1/2}$ and, hence, the number of visits grows as $(t/D)^{1/2}$. The integration over all layers gives an additional multiplier, which grows with time as $\underline{X}(t)\propto(Dt)^{1/2}$. Combining these expressions one obtains $\langle Y^2(t)\rangle\propto D^{-1/2}t^{3/2}$, i.e., the result of Eq. (18a). Let us note that the ASD is *inversely proportional* to $D^{1/2}$, where D is the diffusion constant for the motion along the X axis. Such a dependence is rather evident—with an increase of D both the mean-square displacement along the X axis and the number of distinct layers visited increase. Correspondingly, $N(X,t)$ decreases, which, due to the form of Eq. (39), leads to a decrease of the ASD along the Y axis due to the decrease of the prefactor. This explains the enhancement in the bead's displacement in Eqs. (36) and (37).

After having seen how the change in D modifies the ASD along the Y axis let us examine now how the ASD is

influenced [19,20] by the change in the time dependence of $\overline{X^2(t)}$; for this we consider a random process $X(t)$, characterized by a mean-square displacement growing as t^β with $\beta \leq 2$. In this case the behavior of the ASD can be estimated along the same lines as in the case of conventional diffusive motion. The number of distinct layers visited grows as $t^{\beta/2}$ and, therefore, the number of visits of a given layer grows in proportion to $t^{1-\beta/2}$. The integration over all layers gives a factor $t^{\beta/2}$. Thus, we find that in this case the dynamical exponent is changed, $\langle Y^2(t) \rangle \propto t^{2-\beta/2}$. The exponent β measures the compactness of the trajectory $X(t)$. With an increase of β the range of $X(t)$ grows and, therefore, the number of layers visited by $X(t)$ grows, resulting in a decrease of both $N(X,t)$ and the ASD along the Y axis. Conversely, with the decrease of β the trajectory $X(t)$ becomes more compact and the number of distinct layers visited decreases, while the number of visits to a given layer gets larger. Consequently, for $\beta < 1$ the ASD $\langle Y^2(t) \rangle$ shows a stronger time dependence than for $\beta = 1$ and a weaker time dependence for $\beta > 1$. In our model the mean-square displacement of a tagged bead along the X axis is given by Eq. (23), i.e., it exhibits subdiffusive behavior with $\beta = \frac{1}{2}$. Compared to the trajectory of a WP the bead's trajectory in the X direction is more confined; thus, the number of distinct layers visited is less than that for the WP. As a consequence, the bead's ASD in the Y direction is larger and follows $\overline{\langle Y^2(t) \rangle} \propto t^{2-\beta/2}$ with $2-\beta/2 = \frac{7}{4}$.

We note that the heuristic arguments presented here to discuss the influence of anomalous diffusion along the X axis on the ASD in the Y direction can be supported by a strict analysis along the lines of Refs. 19 and 20. In Refs. 19 and 20 the following generalization of the MdM model has been examined: Suppose that in the (X,Z) plane the particle is allowed to move only in restricted regions, which form a fractal substrate of dimension d_f. The diffusion of a particle on this substrate is anomalous and is characterized by an exponent d_ω; then the growth of $\overline{X^2(t)}$ and $\overline{Z^2(t)}$ follows t^{2/d_ω}. On the basis of the direct calculation of $N(X,t)$, performed for several examples (Sierpinski gaskets and ultrametric spaces), it was found [19,20] that the ASD along the Y axis grows as

$$\overline{\langle Y^2(t) \rangle} \propto t^{2-d_f/d_\omega}, \quad d_f < d_\omega . \tag{40}$$

Keeping in mind, of course, that we consider the motion of a tagged bead of a polymer, while Refs. 19 and 20 concern the motion of a single particle with anomalous diffusion properties, we note that Eq. (40) reproduces our results for Q_d (δ-correlated fields). In fact, for case A $d_f = 1$ and $d_\omega = 4$ [Eq. (23)] we recover Eq. (31a). If we set for case B $d_f = 2$ and $d_\omega = 4$, we arrive at Eq. (32a).

We complete this paper with two remarks.

The exponents which characterize the growth of the ASD in our Eqs. (31) and (32) are model dependent. It is tempting to analyze the behavior of other systems in MdM fields; as examples one may focus on models with finitely extensible springs, models with internal viscosity, or the Verdier-Stockmayer model with fixed interbead distances. Although other behaviors of the ASD as a function of time than our results Eqs. (31) and (32) may appear, we argue that our qualitative result, namely, that the ASD of a bead grows stronger with time for long rather than for short chains, will still hold. Also one can expect that for models where the connections of the beads into the chain are more rigid the effect of the enhancement on the ASD would be stronger. To see this, let us note that the ASD in Eqs. (31) and (32) turns out to be proportional to a power of K; K on the other hand is a measure of rigidity in our model. Correspondingly, an increase in K leads to a larger ASD of the bead.

The Rouse model used in the present work is quite simplified because the beads are presumed to be subject to independent forces, whereas in practice the different parts of the chain will be coupled hydrodynamically [1,8]. Due to hydrodynamic effects the chain may move in an inert fluid more like a solid ball rather than a coil [1,8]. On the other hand, it is well known that the presence of externally applied flows results in the stretching of a chain. Hence, the competition between these effects and their influence on the polymer motion in layered random flows represents an open problem, worth further investigations.

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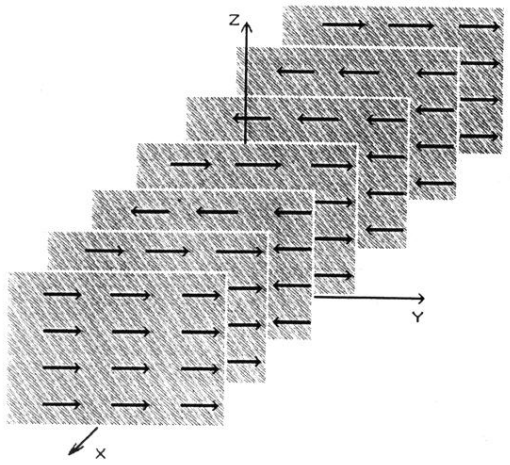

FIG. 1. Case A. The force orientation is a random function of the X variable only and is constant within each layer.

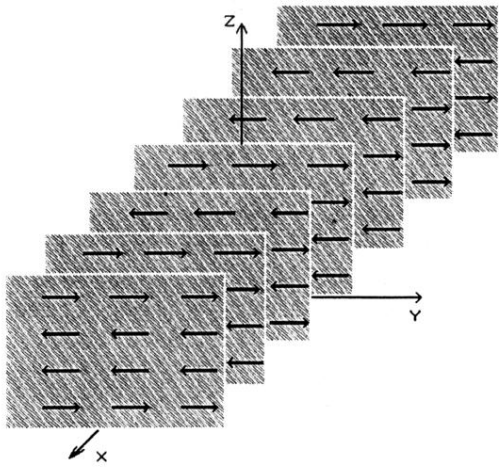

FIG. 2. Case B. The force orientation fluctuates both along the X and Z directions.